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Widely Applicable Insights into the Time Evolution of Massive Gaussian Random Particle Distributions

In cosmology, the Zel'dovich approximation is broadly used to describe how massive particles evolve to form cosmic structures. Insights gleaned from studying structure formation on cosmological scales have now been shown to also apply to smaller-scale physical systems, including Rydberg gases and protoplanetary disks, beginning from a near-uniform Gaussian random distribution of particles.

By Ethan R. Siegel

Original research: Sara Konrad, Matthias Bartelmann, On the asymptotic behaviour of cosmic density-fluctuation power spectra, *Monthly Notices of the Royal Astronomical Society*, Volume 515, Issue 2, September 2022, Pages 2578–2590, https://doi.org/10.1093/mnras/stac1795

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New Ground article reviewed by: Sara Konrad and Matthias Bartelmann

In the fields of physics and astronomy, there are a few widely used approximations that appear to work more successfully, and under a broader range of conditions, than one might initially suspect. The Drude model, for example, accurately describes the collision rates of particles traveling through rarefied astrophysical plasmas just as well as those of particles within densely packed solid-state lattices. Density wave theory, originally intended to explain the spiral arm structures of stars within galaxies, also explains the ordering of hydrogen clouds and dust bands within those arms, as well as numerous properties of circumplanetary rings, such as those surrounding Saturn. A recent publication by Sara Konrad and Matthias Bartelmann now shows that there is yet another example: the Zel'dovich approximation, which is one of the simplest assumptions one can make in Lagrangian perturbation theory. It works spectacularly well in terms of describing the time evolution of Gaussian random fields – better than we'd have any right to expect.

Universal statistical behaviors of cosmic structures on small cosmic scales

Konrad and Bartelmann pursue a kinetic field theory approach using the Zel'dovich approximation and prove that the cosmic density fluctuation power spectrum develops an asymptotic tail that depends solely on the number of spatial dimensions. Since the number of dimensions remains constant, the fluctuation power encoded on cosmologically small distances, corresponding to high wave numbers, also remains constant. Combining this with earlier results that quantified the effects of particle interactions in regimes also beyond the Zel'dovich approximation, the authors conclude that cosmic structures will display universal statistical behaviors on small cosmic scales.

Their finding thus has implications that extend far beyond the field of physical cosmology. Originally, when it came to the formation of cosmic structures, the first approach taken was Eulerian: you would have a near-uniform background of massive particles with initial, small-magnitude overdensities and underdensities superimposed atop it. Any massive "test" particle within this model universe will change position over time, with gravity drawing masses into the overdense regions and away from the underdense ones. In 1969, Yakov Zel'dovich introduced a Lagrangian approach to this problem. Instead of expanding the density contrast – the deviation of density relative to mean density – in a perturbative series, as was done in Eulerian approaches, in the Zel'dovich method particle trajectories are perturbatively expanded in Lagrangian coordinates. This seemingly pure mathematical detail offers a number of clear advantages: among others, it can successfully describe even large-magnitude density contrasts.

The Zel'dovich approximation, in its simplest form, states that if you have a Gaussian random field – that is, a density field with imperfections on various scales that follow a Gaussian distribution in their magnitude – you can describe how perturbations grow and how complex structures form. Even when the departures from the mean density are no longer small, the Zel'dovich approximation is uncannily accurate for large cosmic scales. Its applications extend far into the nonlinear regime of the power spectrum and well beyond the originally intended use for cosmological structure formation. In fact, the Zel'dovich approximation can provide rich insights into the evolution of any Gaussian random field.

Analytical approach to explaining observed small-scale behavior

Intricate cosmological N-body simulations, following the trajectories of individual test particles, must calculate how structure formation proceeds under significant departures from uniformity. Nevertheless, features such as sheets, filaments and voids are all reproduced remarkably well by the Zel'dovich approximation, even on intermediate cosmic scales. This has been known for several decades, with standard explanations for the approximation's success asserting that curl-free particle streams will inflow into any overdense region, and when the shortest axis of the triaxial ellipsoid that results first collapses due to gravitation, the ellipsoid will "pancake," producing a bound structure that corresponds to e.g. a bound galaxy. It has also often been noted that when one computes the power spectrum $\mathscr{P}$ of the density field, the behavior on small distance scales asymptotes in a fashion that's proportional to k^{-3}, where k is the wave vector used in momentum space. But we've never understood why this small-scale behavior always arises.

This is the key insight derived in the new publication by Konrad and Bartelmann. Using kinetic field theory for cosmic structure formation, they analytically determine – even for regimes beyond the Zel'dovich approximation – that at large wave numbers, corresponding to small distance scales, the power spectrum always falls off as k^{-3}, irrespective of the sound horizon scale, redshift, underlying cosmological model and magnitude of the initial fluctuations, and largely independent of the type of dark matter. The exponent itself, -3, arises only from the number of spatial dimensions present in this physical system: 3.

Applicable to any initially near-uniform distribution of matter with Gaussian imperfections

This insight has important consequences for the dimensionless power spectrum, $k^3\mathscr{P}$, which

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illustrates key aspects of structure formation. Instead of experiencing an exponential suppression of small-scale structures, the product, $k^3\mathscr{P}$, remains constant for ever-diminishing distances as $k \to \infty$. This finding could help explain how and why small-scale structures began forming so early on in our cosmic history.

This property of power spectra resulting from an initially Gaussian random field, however, applies to much more than cosmological structure formation. There are a great many systems, both physical and mathematical in nature, that can be characterized as arising from initial conditions that are Gaussian and random in nature. Any initially near-uniform distribution of matter with Gaussian imperfections, including smooth protoplanetary disks, cold Rydberg gases, and systems possessing Brownian motion properties, along with a variety of other stochastic processes, should all exhibit the same behavior on small distance scales in their power spectra.

It’s remarkable that such a simple set of assumptions – that there are straight particle trajectories that don’t have a curl, and that they arise from a Gaussian random field – can lead to what appears to be a universally applicable result. You might be surprised at what powerful conclusions you can draw, even far into the nonlinear regime, from these newly derived consequences arising from the Zel’dovich approximation.

After all, what’s most important in any scientific field is not what type of system a set of equations was intended to describe, but rather what it ultimately proves useful for calculating. With this new insight into the Zel’dovich approximation’s applicability to the large-*k* end of the power spectrum, a whole new suite of potential applications may be opening up before our very eyes. ●

The researchers

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